I. (20 Points) Answer by <u>True</u> or <u>False</u>. To obtain complete grade, show all the calculations and proofs where needed:

1) If the square matrices A and B are symmetric then the matrix (A + B) is symmetric (……..).
2) Det(4A)= 4Det(A) for all (4×4) matrices (……..).
3) The matrices $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$ are inverses (…….).
4) The determinant of any diagonal $(n \times n)$ matrix is the product of the entries on the main diagonal (……..).
5) The matrix $\begin{bmatrix} k^2 & 1 & 4 \\ k & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible for all <u>positive</u> constants k (……..).
6) The vectors $\vec{v} = (a,b)$ and $\vec{w} = (-b,a)$ are orthogonal vectors (……..).
7) If $\|k\vec{u}\| = k\|\vec{u}\|$, then $k \geq 0$ (……..)
8) The determinant of $A = \begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \end{vmatrix}$ is 0 (……..).
9) The system $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is inconsistent (……..)
10) If a matrix A is in reduced row echelon form, then at least one of the entries in each column must be 1 (…….).

II. (10 Points) Find the Area of the triangle having vertices $A(0;2;3)$, $B(1;0;1)$ and $C(2;1;0)$. Having the area, deduce the length of the altitude from vertex *C* to side *AB*.

III. (15 Points) Consider two distinct numbers, *a* and *b*. We define the function:

$$f(t) = \det\begin{bmatrix} 1 & 1 & 1 \\ a & b & t \\ a^2 & b^2 & t^2 \end{bmatrix}$$

a) Show that *f*(*t*) is a quadratic function (i.e. of degree 2). What is the coefficient of t^2?
b) Explain why *f*(*a*) = *f*(*b*) = 0. Conclude that $f(t) = k(t-a)(t-b)$, for some constant *k*. Find *k* using your work in part (a).
c) For which values of t is the matrix invertible?

IV. (20 Points) Let $\vec{u} = (3;2;1)$, $\vec{v} = (5;-3;4)$ and $\vec{w} = (1;6;-7)$. Compute the following:

a) $\|\vec{u}\|(\vec{v} \bullet \vec{w})$
b) $\left\|proj_{\vec{u}}\vec{v}\right\|$
c) $\|3\vec{u} - 5\vec{v} + \vec{w}\|$
d) $\vec{u}.\left(\vec{w} \times \vec{v}\right)$
e) $\vec{u} \times (\vec{v} - 2\vec{w})$

V. (20 Points) Let the vectors $\vec{u} = (1;-1;3)$, $\vec{v} = (1;-2;-7)$ and $\vec{w} = (2;3;4)$. Find scalars c_1, c_2, and c_3 such that $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = (8;1;10)$

VI. (15 Points) Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

a) Show that *A* can be written as $aI_3 + bB$, where *a*, *b* are scalars to determine, and I_3 is the identity matrix (3 × 3).
b) Determine B^2.
c) Determine A^2 and verify that $A^2 = 7A - 10I_3$
d) Determine A^{-1}